Games in Normal and Satisfaction Form for Efficient Transmission Power Allocation Under Dual 5G Wireless Multiple Access Paradigm

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***Abstract*— In this paper, to exploit the challenges and potential offered by the simultaneous use of non-orthogonal multiple access (NOMA) and orthogonal frequency division multiple access (OFDMA) transmission options in future 5G wireless systems, we aim at the proper modeling and transformation of the uplink power allocation problem. In particular, in this setting, each user has two degrees of freedom in the decision making process, namely its overall transmission power level, and the corresponding power investment to the OFDMA and/or NOMA based transmissions. The resulting multi-variable power allocation problem is treated and solved under three different perspectives, namely: 1) Games in Normal Form and Nash Equilibrium (NE); 2) Optimization techniques targeting system social welfare through a centralized optimal solution; and 3) Games in Satisfaction Form and Efficient Satisfaction Equilibrium (ESE). Based on these approaches, different solutions and stable operation points are identified and their properties are analyzed. An in depth evaluation and comparison of the various obtained outcomes is achieved, via modeling and simulations. The focus is placed on the impact and the interplay of the NOMA specific features, including the potential over-exploitation of the available bandwidth, the fairness in accessing it, and the interference treatment. It is also shown that, using the satisfaction form games for the users to converge to the ESE, provides an efficient and promising user-centric modeling approach to the power allocation problem, as the system adapts to the users' application needs, while at the same time eliminates a significant amount of interference.**

***Index Terms*— Resource optimization, dual multiple access technology, 5G systems, game theory, satisfaction equilibrium.**

I. Introduction

MOBILE data traffic and the number of connected devices are growing at an unprecedented pace, with this trend expected to further intensify via the deployment of 5G and beyond networks [1]. However, the desire to fully realize the capabilities of 5G technologies and features, while accounting for spectrum scarcity and efficiency, creates the pressing need to examine the potential of 5G ready radio access technologies in optimizing resource allocation.

In that respect, non-orthogonal multiple access (NOMA) is one of the most promising radio access techniques in next-generation wireless communications, offering a set of desirable potential benefits, such as enhanced spectrum efficiency, reduced latency, resource usage fairness, and massive connectivity [2]. The fundamental idea of NOMA is the multi-user spectrum sharing within a resource block through power-domain multiplexing, capitalizing on the exploitation of channel gain difference among users. By allowing multiple users to be superimposed on the same resource NOMA, on the one hand prevents the underutilization of system bandwidth, while on the other hand leads to interference for such systems [3], [4]. Advanced physical layer and multi-user detection techniques, such as Successive Interference Cancellation (SIC), are then, applied at the receiver to decode the received superimposed signal and deal with the interference problem [5].

However, since the principle of NOMA allows multiple users to be superimposed on the same resource, this leads to interference for such systems. On the other hand, orthogonal frequency division multiple access (OFDMA) assigns to the individual users different subcarriers which are orthogonal to each other [6]. Accordingly, the key benefits of OFDMA include multi-user diversity gains and elimination of intra-cell interference, thus leading to lower complexity and more predictable performance. Consequently, while variations of NOMA are receiving increasing attention recently [7], OFDMA is still expected to remain an integral part of the forthcoming 5G networks [8]. A more thorough presentation of the differences of the OFDMA versus the NOMA technique can be found in [9].

With the emergence of smart devices with dual transmission access capabilities being reality [10], spectrum sharing techniques can also become available under different access technologies simultaneously. An interesting paradigm involves users dynamically adjusting their transmission between (a) the interference free but limited in terms of throughput OFDMA,

Manuscript received June 19, 2020; revised October 12, 2020 and February 25, 2021; accepted June 29, 2021; approved by IEEE/ACM Transactions on Networking Editor D. Cansever. This work was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) through the "1st Call for H.F.R.I. Research Projects to support Faculty members and Researchers and the procurement of high-cost research equipment grant" under Project HFRI-FM17-2436. *(Corresponding author: Symeon Papavassiliou.)*

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This article has supplementary downloadable material available at https://doi.org/10.1109/TNET.2021.3095351, provided by the authors.

Digital Object Identifier 10.1109/TNET.2021.3095351

and (b) NOMA, where the entire available spectrum can be exploited, however it has to be shared with the rest of users.

Within this emerging setting, in this paper, we aim to lay the foundations for the design of a solid theoretical resource allocation framework based on the principles of game theory and the novel concept of satisfaction games. Each user has two degrees of freedom in the decision making process, namely its overall transmission power level, and the corresponding power investment (split) to the OFDMA and NOMA based transmissions. In this paper, three different types of resource allocation formulations are examined, reflecting different perspectives and objectives, while concluding to different stable operation points. Namely, we analyze and compare: (i) Distributed resource allocation, where the users act as utility maximizers; (ii) Centralized resource allocation aiming at maximizing the overall system's social welfare, expressed as the summation of all user individual utilities, and (iii) Satisfaction-aware resource allocation towards satisfying the users' minimum Quality of Service (QoS) prerequisites. The latter targets the objective of improving user energy awareness and efficiency, while increasing system capacity in terms of satisfied users.

A. Related Work & Motivation

Several efforts have been performed in the recent literature to deal with the uplink power control problem in NOMA-based 5G networks. In [11], a power control problem is formulated as a non-cooperative game among the users of a heterogeneous network, consisting of macro-cell and femtocell users, aiming at the maximization of their energy efficiency. The non-cooperative game is solved in a centralized manner by using convex optimization techniques to determine the unique Nash Equilibrium (NE). In [12], an uplink power control mechanism in NOMA networks is introduced, while simultaneously controlling the number of users that share the same bandwidth's subchannel in order to reduce the complexity of multi-user detection at the receiver. A different approach is followed in [13], where the authors formulate a multi-variable non-cooperative game among the users, who aim to determine their optimal transmission data rate and customized price. The problem is solved based on the theory of S-modular games and a NE point is obtained, that allows each user to determine its optimal transmission power. The power control problem over NOMA-based licensed and unlicensed bands is studied in [14], [15]. In that line of work each user, assuming fixed transmission power, determines only the power investment (portion) to the licensed and unlicensed band-based communication, while capturing the users' behavioral characteristics and preferences through the adoption of Prospect Theory.

Considering the availability of both NOMA and orthogonal multiple access (OMA) schemes (such as OFDMA [16]), in [10] a centralized power control problem is formulated, aiming at maximizing the system's sum rate and solved via the successive convex approximation technique, to determine each user's transmission scheme (either NOMA or OMA), and corresponding transmission power. In this preliminary research work however, the problem is studied under the assumption of exclusive use of either NOMA or OMA technique, without enabling the users to jointly exploit the benefits of both techniques. The latter limitation and issue is treated in [17], where the problem of resource allocation within the flexible dual access technology paradigm is studied, under the framework of Common Pool Resource (CPR) games and Prospect Theory. Despite the promising results obtained in this research work in terms of spectrum efficiency utilization, it is assumed that each user transmits with a fixed overall transmission power and can only control its power investment (split) to the NOMA and OFDMA transmission. However, this assumption reduces the solution's flexibility in terms of controlling overall power consumption, thus limiting its applicability and effectiveness.

The synergy between the resource allocation problem in wireless networks, and in particular power control, and game theory is well established in the literature [18]–[22]. Nevertheless, in the overwhelming majority of the existing in the literature power control approaches, the users act as utility maximizers aiming at maximizing their perceived satisfaction from the resource allocation process in a selfish and greedy manner. Therefore, for spectrum and energy efficiency mainly considerations, the framework of games in satisfaction form [23], is gaining great attention in the recent literature towards addressing the problem of resource allocation in 5G networks [24]. Under this perspective, the users participate in the resource allocation process and determine the most cost efficient strategies in order to satisfy their minimum QoS prerequisites, rather than maximizing some objective function. In [25], a learning iterative mechanism is introduced to determine the Satisfaction Equilibrium (SE), that is, to determine the users' transmission powers at the point where their minimum prerequisites are fulfilled. A theoretical analysis of various equilibrium points of the games in satisfaction form is provided in [26], studying the efficiency of the uplink power control problem in 5G networks, both from the users' and the system's perspective.

B. Contributions and Outline

To the best of our knowledge, our work is the first one in the literature that aims at removing the aforementioned assumptions and treating the emerging challenges. In particular, it aims at transforming the holistic uplink power control problem, in order to consider different perspectives and transmission options in the user decision making process, under the reality of the dual access technology paradigm in future wireless networks, capitalizing on the potential simultaneous use of OFDMA and NOMA transmission options. Considering a user utility function based on the Shannon capacity formula, each user aims at simultaneously selecting its optimal transmission power and respective splitting factor (i.e., investment of its transmission power between the OFDMA and the NOMA), given certain objectives, constraints and the state of the network, as reflected by the strategies of the other users and the adopted pricing schemes. We refer to this joint decision making problem as power allocation problem. This multi-variable problem is investigated from three different perspectives, motivating and calling for the following treatments: (i) Games in Normal Form and NE, (ii) Optimization techniques targeting system social welfare through a centralized optimal solution, and (iii) Games in Satisfaction Form and Efficient Satisfaction Equilibrium (ESE).

In particular, initially considering users acting as utility maximizers and thus reflecting an egoistic behavior, we model the aforementioned setting as a game in normal form and we prove that the Best Response Dynamics (BRD) converges to the NE of the game. We further introduce a linear - with respect to the user power investment - cost function in each user's utility, which can ameliorate the aforementioned selfish behavior and make the system more efficient. In this manner, a more socially desirable output is obtained. Furthermore, a distributed algorithm is provided that efficiently solves the maximization problem of finding a user's Best Response (BR), by testing the prospective local optima. A sufficient condition that guarantees that a user's BR does not lie at its maximum transmission power is provided as well.

Subsequently, we revisit and study the corresponding power allocation problem within the novel setting of dual access transmission options from the perspective of maximizing the system social welfare, that is maximizing the summation of the users' utilities. This is realized, through a centralized approach by assuming that a central authority has full knowledge of all the required information for every user. Surprisingly enough, it turns out and that in this case the optimal solution occurs when the users transmit with their maximum possible power. This is due to the nature of NOMA and the operation of the SIC technique, and its impact on the optimization problem under consideration, as explained later in the paper.

Towards providing a more efficient and effective solution to the power allocation problem under consideration, while maintaining its distributed and user-centric nature, we undertake the perspective of games in satisfaction form. In this outlook, every user aims at investing sufficient resources towards satisfying its QoS prerequisites, rather than purely maximizing its utility. Specifically, we redesign the BR functions to fit this framework and we prove that the BRD converge to the unique ESE under one assumption that, interestingly enough, turns out to be necessary and sufficient condition for the existence of the ESE. We provide a solution to the problem of minimizing the transmission power of each user subject to meeting its QoS prerequisites, in order for it to find its unique BR. In this way, a distributed algorithm is devised, which when deployed by every user the system efficiently converges to the unique ESE.

Based on the above theoretical foundations, we study in detail and demonstrate, how a network provider can effectively employ different pricing schemes, in order to exploit the NE of the game, towards augmenting the capacity of the network and limiting the total interference. We observe and conclude that, although the social welfare optimal solution results in the application of no pricing in the system, charging the users with cost that is commensurate with their transmission powers, is imperative to reduce the interference, improve fairness in resource allocation, and consequently result in more efficient NE operation points. This observation, enables and is enabled by the nature of NOMA and in particularly the SIC feature. In principle, at the absence of any pricing mechanism, users far from the BS (i.e., bad channel gains) may over-exploit the SIC technology in their NOMA-based transmission, as they remain practically unaffected by the strategies of the others. This in turn increases the interference sensed in the NOMA-based transmission by the users with good channel gains.

It is also shown that the potential over-exploitation of the system by the users with bad channel gains, can be overcome in the case of the centralized solution where overall system social welfare maximization is targeted, by having those users to transmit only in the OFDMA-based channel. This however introduces unfairness among the users in terms of accessing the available NOMA-based bandwidth, and at the same time does not utilize the system resources and the dual technology benefits at their full potential. To overcome these issues, it is argued and demonstrated that, when the framework of games in satisfaction form is instead adopted, the overall resource allocation becomes not only more user centric - as the user QoS prerequisites are inherently taken into consideration in a more personalized manner - but also significant improvements from the system point of view are obtained as well.

The remaining of the paper is organized as follows. In Section II some background information about the games in normal and satisfaction form is provided, while in section III the dual access technology paradigm and model is introduced, and the corresponding utilities are formally defined. In Section IV a distributed resource allocation approach is presented based on the normal form games and the NE concept, and subsequently Section V contains the presents a solution targeting the overall system's social welfare optimization. In Section VI the power allocation problem is reformulated and studied through the games in satisfaction form, aiming at a satisfaction-aware resource allocation approach. In Section VII a detailed numerical evaluation of the performance of the proposed power allocation framework is provided, through modeling and simulation, illustrating the operation, features and benefits of each one of the proposed approaches. Finally, Section VIII concludes the paper.

II. Games in Normal and Satisfaction Form

In this section, we provide some game theoretic definitions and notation that will be used throughout the paper.

A. Games in Normal Form

A game in normal form is defined as $G = (K, \{A_k\}_{k\in K}, \{u_k\}_{k\in K})$, where $K = \{1, \dots, |K|\}$ represents the set of players, A_k is the action set of player $k \in K$, $u_k(a_k, \boldsymbol{a}_{-k})$ represents player k's payoff (i.e., utility function). An action profile is denoted by a vector $\boldsymbol{a} = (a_1, \dots, a_{|K|}) \in A$, where $A = A_1 \times \cdots \times A_k \times \cdots \times A_{|K|}$.

Definition 1: An action profile $\boldsymbol{a}^{\boldsymbol{NE}}$ is a *Nash Equilibrium* (NE) point for the game $G = (K, \{A_k\}_{k\in K}, \{u_k\}_{k\in K})$ if

$$u_k(a_k^{NE}, \boldsymbol{a}_{-k}^{NE}) \geq u_k(a_k', \boldsymbol{a}_{-k}^{NE}) \quad \forall a_k' \in A_k,\ \forall k \in K \quad (1)$$

Mechanisms in which the players seek to maintain payoffs that are above a given threshold, instead of maximizing them, can be modeled through games in satisfaction form.

B. Games in Satisfaction Form

A game in satisfaction form is defined as $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$, where $f_k(\boldsymbol{a}_{-k}) = \{a_k \in A_k :$

$u_k(a_k, \boldsymbol{a}_{-k}) \geq u_k^{thr}\}$ determines the set of actions of player k that allows the satisfaction of the minimum QoS prerequisites or, in other words, actions that allow its payoff to be above a given threshold value u_k^{thr}, considering the actions $\boldsymbol{a}_{-k}$ played by all the other players [24]. Games in Satisfaction Form arise as a powerful tool for analyzing systems for which, although developing a competitive nature, the motives of the players depend primarily on reaching a threshold in their pay-off, instead of selfishly maximizing it. Relaxing the maximization assumptions essentially we enlarge the set of feasible strategies since, instead of restricting ourselves to solutions of global optimum, we extend the solution space to a broader set. In a practical setting, this would lead to important resource savings and to an increase in the corresponding system capacity, in terms of satisfied users.

Definition 2: An action profile $\boldsymbol{a}^+$ is a *Satisfaction Equilibrium (SE)* for the game $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$, if

$$a_k^+ \in f_k(\boldsymbol{a}_{-k}^+), \quad \forall k \in K \tag{2}$$

It should be emphasized that there could exist multiple action vectors $\boldsymbol{a}^+ = (a_1^+, \ldots, a_{|K|}^+)$ satisfying the player's minimum QoS prerequisites, some of which are of particular interest. With those equilibria, the characterization of the feasible region of the strategy profiles, where everyone is satisfied, is achieved. A representative example of a subset of SEs that might present interest is the *Efficient Satisfaction Equilibrium* (ESE) where each player of the system achieves its minimum QoS prerequisites via being simultaneously penalized with the minimum cost. To capture the notion of the players' penalty and effort associated with a given action choice, the concept of the cost function for each player is introduced. For all $k \in K$, the cost function $c_k : A_k \rightarrow [0, 1]$ satisfies the following condition: $c_k(a_k) < c_k(a'_k)$, $\forall (a_k, a'_k) \in A_k^2$, if and only if, a_k requires a lower effort by player k than action a'_k.

Definition 3: An action profile $\boldsymbol{a}^*$ is an *ESE* point for the game $\hat{G}$, with cost functions $\{c_k\}_{k\in K}$, if

$$a_k^* \in f_k(\boldsymbol{a}_{-k}^*), \quad \forall k \in K \tag{3a}$$

$$c_k(a_k) \geq c_k(a_k^*), \quad \forall k \in K,\ \forall a_k \in f_k(\boldsymbol{a}_{-k}^*) \tag{3b}$$

III. Dual Access Technology Paradigm & Model

In this paper we consider the coexistence of OFDMA and NOMA-based communication in 5G wireless networks. Each User Equipment (UE)[1] has a dual communication interface to transmit data either via using NOMA or OFDMA technique. The frequency band operating over OFDMA is split into resource blocks, where only one UE transmits exclusively per resource block, thus, the intracell interference from those transmissions, is eliminated. The frequency band operating over the NOMA technique is accessed with equal rights and priority by all the UEs.

Each UE's goal is to opportunistically choose its transmission power levels and determine in an autonomous manner its optimal transmission power split over the NOMA and the OFDMA operating bands, to fulfill its QoS prerequisites. Specifically, assuming that each UE k, $k \in K$ picks a transmission power p_k, which can be invested to the OFDMA and NOMA based transmissions, the percentage of transmission power investment to the NOMA transmission is x_k, $x_k \in [0, 1]$, thus, the corresponding transmission power is $p_k^N = x_k p_k$, while the transmission power over the OFDMA is the remaining amount, i.e., $p_k^O = (1 - x_k)p_k$.

Let us consider $|K|$ transmitters denoted by index $k \in K$ communicating with a given base station (BS). For all $k \in K$, transmitter k uses a power level $p_k \in A_k$. For each player $k \in K$, p_k^{max} denotes the maximum feasible power level, while h_k denotes the channel gain coefficient between transmitter k and the base station. We study the uplink power allocation game where each user captures its received payoff from each transmission (NOMA or OFDMA) with a utility function - following Shannon capacity formula - as below.

$$u_k(p_k, \boldsymbol{p}_{-k}) = W \log_2(1 + \frac{p_k h_k}{\sigma^2 + J_k})[\frac{bps}{Hz}] \tag{4}$$

where σ^2 denotes the Additive White Gaussian system's Noise variance at receiver k, W is the considered bandwidth to the user's transmission, and J_k denotes the interference sensed by player k. As mentioned before, user's k interference in the OFDMA channel will be eliminated and its payoff will be independent of the transmission powers of the others. Thus, in the OFDMA scenario, we have $J_k = 0$. However, in the NOMA frequency band, player k's sensed interference will be $J_k = \sum_{j>k} p_j^N h_j$, i.e, it will sense the interference only from the transmissions of the users with worse channel gains. Thereby, for notation purposes, we assume that the users in the set K are ordered according to their distance from the BS with player $|K|$ being the furthest, thus, $h_1 > h_2 > \cdots > h_{|K|}$.

Following the previous model, we construct the total obtained actual utility of player k from its dual transmission in the NOMA and OFDMA- based bands, as follows:

$$U_k(p_k, x_k; \boldsymbol{p}_{-k}, \boldsymbol{x}_{-k}) = u_k^O(p_k, x_k) + u_k^N(\boldsymbol{p}, \boldsymbol{x}) - c_k(p_k) \tag{5}$$

where, (a) the first term expresses the obtained utility from the OFDMA-based transmission, $u_k^O(p_k, x_k) = A \cdot \log_2(1 + \frac{h_k p_k^O}{\sigma^2})$ (obtained by appropriate adoption of the aforementioned Eq. 6), (b) the second term expresses the corresponding utility obtained from the NOMA-based transmission, $u_k^N(\boldsymbol{p}, \boldsymbol{x}) = C \cdot \log_2(1 + \frac{h_k p_k^N}{\sigma^2 + \sum_{j>k} p_j^N h_j})$ (obtained by appropriate adoption of the aforementioned Eq. 6) and, finally, (c) the third term is a *cost function* that maps the user's transmission power levels to a real number, and represents the users' need to converge to an energy efficient outcome. It is noted that A [Hz] represents the bandwidth of an OFDMA channel allocated to user k, and C [Hz] is the total shared bandwidth among the users transmitting in the NOMA-based frequency band.

IV. Dual Access Technology Option as a Normal Form Game

Following the previous analysis such a competitive communication environment can be modeled via a normal form game that is defined as $G = (K, \{A_k\}_{k\in K}, \{U_k\}_{k\in K})$, where $K = \{1, \ldots, |K|\}$ represents the set of users in the examined network and $U_k(\boldsymbol{a}_k, \boldsymbol{a}_{-k})$ represents player k's payoff (i.e.,

[1]In the paper, the terms User Equipment (UE), transmitter, user, and player are used interchangeably.

utility function) that is given by Eq. 5. A_k is the set of all the available strategies of player $k \in K$ and it consists of the two variable tuples: $A_k = \{\boldsymbol{a}_k = (p_k, x_k) \mid 0 \le p_k \le p_k^{max} \;\; and \;\; 0 \le x_k \le 1\}$. The users' competitive behavior to transmit their data over the OFDMA and NOMA bands is the driving factor to formulate their interactions as a normal form game. Since each user has infinitely many strategies to choose from, i.e., the game is infinite, the existence of Nash Equilibrium is not guaranteed in general. Therefore, our ultimate goal is to examine the existence of a Nash Equilibrium point and introduce a mechanism that enables the users to converge to the NE, that is, all maximizing simultaneously their individual utilities.

A. Nash Equilibrium and Best Response Dynamics

In this section, we prove the existence of a NE in the game G and we show that the Best Response Dynamics (BRD) converge to such a point. The first step towards this goal is the introduction of the Best Response (BR) of a player k in a normal form game, which is the strategy that maximizes its utility function given the strategies of the others $\boldsymbol{a}_{-k}$, i.e., $BR_k(\boldsymbol{a}_{-k}) = \{\boldsymbol{a}_k \in A_k : \boldsymbol{a}_k = \arg\max_{\boldsymbol{a}_k \in A_k} U_k(\boldsymbol{a}_k, \boldsymbol{a}_{-k})\}$. The BRD is defined as the behavioral rule in which each user chooses its BR whenever it is to respond to changes in the strategies of the others [27].

Clearly, U_k is a continuous function, defined on the compact set $[0, p_k^{\max}] \times [0,1]$, and therefore attains a maximum value, given $\boldsymbol{a}_{-k}$. As it is later proven, U_k has a unique critical point. Thus, assuming that if the maximum happens to belong in the boundary then it is also unique, suffices to ensure that the maximum will always be unique. Note that this maximum is equivalent to k's BR, i.e., $BR_k(\boldsymbol{a}_{-k})$.

Proposition 1: a) Game G possesses a NE and b) In game G, the BRD converges to a NE of the game.

Proof: Let BR_k^t be the Best Response of player k in turn t. Let also t_1 be the turn during which the user $|K|$, i.e., the user having the worst channel conditions, chooses a strategy for the first time. Note that throughout the dynamics, $BR_{|K|}(\cdot)$ will not depend on the strategies of the others as $J_{|K|} = 0$. Therefore, as soon as player $|K|$ plays its BR, it will never deviate from that strategy, i.e., $BR_{|K|}^t = BR_{|K|}^{t_1}$, for all $t \ge t_1$. Let $(p_{|K|}^*, x_K^*) = BR_{|K|}^{t_1}$, then $J_{|K|-1} = p_{|K|}^* x_{|K|}^* h_{|K|}$ will never change from time t_1 onwards. So, if t_2 is the first time after t_1 during which user $|K|-1$ chooses strategy for the first time, then in the turns after t_2 player $|K|-1$ will never deviate from $BR_{|K|-1}^{t_2}$, i.e., $BR_{|K|-1}^t = BR_{|K|-1}^{t_2}$, for all $t \ge t_2$. Continuing in this manner, we conclude that $\exists t^*$: $BR_k^t = BR_k^{t^*}$, $\forall k \in K, \forall t : t \ge t^*$. Therefore, because of the definition of the BR, if we denote the strategy profile $\boldsymbol{a}^{NE}$ as the one with $\boldsymbol{a}_k^{NE} = BR_k^{t^*}, \forall k \in K$ then,

$$U_k(\boldsymbol{a}_k^{NE}, \boldsymbol{a}_{-k}^{NE}) \ge U_k(\boldsymbol{a}_k', \boldsymbol{a}_{-k}^{NE}) \quad \forall \boldsymbol{a}_k' \in A_k, \; \forall k \in K \tag{6}$$

In a nutshell, the existence of the turn t^* proves that the BRD in G will converge at a strategy profile $\boldsymbol{a}^{NE}$ which, due to Eq. 6, will be an NE as well. ■

Notice that Proposition 1 implies that, provided $U_k(\boldsymbol{a}_k, \boldsymbol{a}_{-k})$ admits a unique maximum for every user k and every strategy profile of the others, $\boldsymbol{a}_{-k}$, *the NE will be unique*. It should be also noted that in the general implementation of the BRD, each user is not required to know the distances from the BS and the strategies of the rest of the users. For each user to determine in a distributed and autonomous manner its best response strategy, only the information of his/her personal sensed interference is needed. This is still a challenging and open problem in terms of its implementation, when considering the uplink NOMA-based communication. A possible solution would be the base station to provide the sensed interference to each user via unicasting it, thus significantly reducing the signaling overhead burden from the end-users' side. Moreover if the system knows a priori the ordering of the users depending on their distance from the base station, the convergence time of the BRD to the unique NE can be substantially improved. Specifically, Proposition 1 implies that the first player that should decide on its strategy is player $|K|$, the second should be player $|K|-1$ and so on. In that fashion, the BRD would converge to the unique NE only in $|K|$ turns, i.e., $t^* = |K|$.

In order for the above proposition to be plausible for the system, the calculations of the BRs should be efficient and possible for every user in the network. Therefore, in order to propose an efficient distributed algorithm, we first focus on the problem of maximizing each user's utility (i.e., Eq. 5). In the rest of the paper, we adopt a linear usage-based pricing, i.e., a linear cost function $c_k(\cdot)$ with respect to the transmission power, i.e, $c_k(p_k) = \lambda_k p_k$, where $\lambda_k > 0$ is a personalized user pricing parameter. This parameter can either be set a priori by the user itself to express its own dissatisfaction regarding the consumption of its personal resources, e.g., battery life, or by a centralized authority, as a control parameter, to mitigate the interference and prevent aggressive users from over-exploiting the system.

Proposition 2: The function $U_k(p_k, x_k; \boldsymbol{p}_{-k}, \boldsymbol{x}_{-k})$, given by Eq. 5 and defined on the set $[0, p_k^{\max}] \times [0,1]$, attains its maximum at one of the following points:

1) $(\frac{A}{\lambda_k} - \frac{\sigma^2}{h_k}, 0)$
2) $(\frac{C}{\lambda_k} - \frac{\sigma^2 + J_k}{h_k}, 1)$
3) $(p_k^{\max}, \frac{1}{C+A} \cdot \frac{C h_k p_k^{\max} + C\sigma^2 - A(\sigma^2 + J_k)}{h_k p_k^{\max}})$
4) $(\frac{(A+C)h_k - (2\sigma^2 + J_k)\lambda_k}{\lambda_k h_k}, \frac{C h_k - \lambda_k(\sigma^2 + J_k)}{(A+C)h_k - (2\sigma^2 + J_k)\lambda_k})$
5) $(p_k^{\max}, 0)$
6) $(p_k^{\max}, 1)$

Proof: As already mentioned, U_k attains a maximum value which either occurs at the boundary or at an interior point of its domain. Let $(p_k^*, x_k^*) \in [0, p_k^{\max}] \times [0,1]$ be a local maximum of U_k. Assume first that (p_k^*, x_k^*) does not belong to the boundary of $[0, p_k^{\max}] \times [0,1]$. We proceed by computing

$$\frac{\partial U_k}{\partial x_k} = h_k p_k \left(-\frac{A}{\sigma^2 + h_k p_k (1 - x_k)} + \frac{C}{(\sigma^2 + J_k) + h_k p_k x_k} \right) \tag{7}$$

$$\frac{\partial U_k}{\partial p_k} = h_k \left(\frac{A(1 - x_k)}{\sigma^2 + h_k p_k (1 - x_k)} + \frac{C x_k}{(\sigma^2 + J_k) + h_k p_k x_k} \right) - \lambda_k. \tag{8}$$

We want to determine pairs (p_k, x_k) that simultaneously satisfy both Eq. 7 and Eq. 8 are equal to zero. Let us denote $I_k := (\sigma^2 + J_k)$. We observe that $\frac{\partial U_k}{\partial x_k} = 0$ if and only if

$$C\sigma^2 + C h_k p_k (1 - x_k) = A I_k + A h_k p_k x_k. \tag{9}$$

Furthermore, $\frac{\partial U_i}{\partial p_k} = 0$ if and only if

$$\frac{(C\sigma^2 + Ch_kp_k(1-x_k))\cdot x_k + (AI_k + Ah_kp_kx_k)\cdot(1-x_k)}{(\sigma^2 + h_kp_k(1-x_k))\cdot(I_k + h_kp_kx_k)} = \frac{\lambda_k}{h_k}. \quad (10)$$

Based on Eq. 9, Eq. 10 can be rewritten as $\frac{C}{I_k + h_kp_kx_k} = \frac{\lambda_k}{h_k}$ and therefore

$$p_kx_k = \frac{Ch_k - \lambda_kI_k}{\lambda_kh_k}. \quad (11)$$

From Eq. 11 and Eq. 9, we conclude that

$$p_k^* = \frac{(A+C)h_k - (2\sigma^2 + J_k)\lambda_k}{\lambda_kh_k}, \quad (12)$$

which in turn, using Eq. 11, yields to

$$x_k^* = \frac{Ch_k - \lambda_k(\sigma^2 + J_k)}{(A+C)h_k - (2\sigma^2 + J_k)\lambda_k}. \quad (13)$$

Assume now that (p_k^*, x_k^*) belongs to the boundary of $[0, p_k^{\max}] \times [0,1]$. Clearly, we have $p_k^* \neq 0$. If $x_k^* = 0$, then based on Eq. 8, we have $p_k^* = \min\{\frac{A}{\lambda_k} - \frac{\sigma^2}{h_k}, p_k^{\max}\}$. Similarly, if $x_k^* = 1$, then $p_k^* = \min\{\frac{C}{\lambda} - \frac{\sigma^2 + J_k}{h_k}, p_k^{\max}\}$. Finally, if $p_k^* = p_k^{\max}$ then, using Eq. 7, we conclude $x_k^* = \min\{\frac{1}{C+A}\cdot\frac{Ch_kp_k^{\max} + C\sigma^2 - A(\sigma^2 + J_k)}{h_kp_k^{\max}}, 1\}$. ■

From the proof of the proposition above one should note that, in order for points 1-4 of Proposition 4.2 to be possible local maxima of the utility function, the first coordinate should belong to the interval $[0, p_k^{max}]$ and the second coordinate to the interval $[0,1]$. Notice that the non-negativity conditions for p_k^* and x_k^* provide upper bounds, which will be utilized in the next proposition, for λ_k via Eq. 12 and 13.

Finally in the following proposition, we provide a sufficient condition that guarantees that the critical point given by Eq. 12 and Eq. 13 respectively, is a local maximum of the utility function. Note that the such condition is not necessary in order for the BRD (and thus for the algorithm proposed in the following section) to converge to the NE. However, the following proposition can provide further intuition about when the users obtain greater utilities by choosing strategies from their boundaries. The analogy from the classic problem of the uplink power control would be the sufficient conditions that force the users of the network to transmit with their maximum one [19].

Proposition 3: Suppose that

$$\lambda_k \leq \min\left\{\frac{Ah_k}{2\sigma^2}, \frac{Ch_k}{2\sigma^2 + J_k}\right\}. \quad (14)$$

Then the critical point (p_k^*, x_k^*), whose coordinates are given by Eq. 12 and Eq. 13 respectively, is a local maximum of the function $U_k(p_k, x_k; \boldsymbol{p}_{-k}, \boldsymbol{x}_{-k})$. Furthermore, provided that

$$\lambda_k \geq \frac{(A+C)h_k}{\sigma^2 + (\sigma^2 + J_k) + h_kp_k^{\max}} \quad (15)$$

holds true, we have $(p_k^*, x_k^*) \in [0, p_k^{\max}] \times [0,1]$.

The proof is available in Appendix A.

B. Best Response for Maximization (BRM) Algorithm

Following the analysis from the previous section, we introduce the *Best Response for Maximization* (BRM) algorithm, which suggests holistic dynamics for the users of the system, in order for the system to converge to the NE of the game in a distributed and autonomous manner. First, each user k should (randomly) initialize its strategy, i.e., $\mathcal{D}$. After that, BRM should be executed by each player whenever is its turn to choose a strategy (either sequentially or asynchronously). Note, that although we refer to $\boldsymbol{a}_{-k}$ as the strategies that are chosen by the players except k, player k should know only the strategies of the players $k+1, \ldots, |K|$ in order to execute the algorithm BRM. As stated earlier, if the system knows the ordering of the users, then $|K|$ turns of the BRM algorithm are sufficient in order all the users to converge to the NE. Thereby, in such scenario, due to the $\mathcal{O}(1)$ time complexity required for each user to determine its BR (via the BRM algorithm), the total time complexity of the dynamics would be $\mathcal{O}(|K|)$.

Algorithm 1 Best Response for Maximization (BRM)

$\mathcal{M} \leftarrow 0$; // Represents the maximum utility
$\mathcal{D} \leftarrow \emptyset$; // Represents the maximizers of $U_k()$, corresponding to $\mathcal{M}$, given $\boldsymbol{a}_{-k}$. $\mathcal{D}.p$ denotes the power and $\mathcal{D}.x$ the splitting factor
$\boldsymbol{\delta} \leftarrow \emptyset$;
for $\boldsymbol{\delta}$ `from points 4.2.1 to 4.2.6` **do**
  if $\boldsymbol{\delta} \in [0, p_k^{max}] \times [0,1]$ **then**
    if $U_k(\boldsymbol{\delta}, \boldsymbol{a}_{-k}) > \mathcal{M}$ **then**
      $\mathcal{D} \leftarrow \boldsymbol{\delta}$;
      $\mathcal{M} \leftarrow U_k(\boldsymbol{\delta}, \boldsymbol{a}_{-k})$;
    end if
  end if
end for
play $\mathcal{D}$; // Play the Best Response

V. Dual Access Technology Option for System Social Welfare Optimization

In this section, a centralized approach to the power allocation problem is examined, aiming at determining the users' optimal overall transmission power, as well as the power investment in their NOMA and OFDMA-based transmissions, i.e., $p_k^*, x_k^*, 1 - x_k^*$ respectively. The corresponding problem aims at maximizing the system's social welfare, i.e., the summation of the users' utility functions (Eq. 5). Note that, in this formulation, the central authority might tune the pricing parameter, λ_k, in order to maximize the aforementioned summation and, thus, in this section, it is considered as a control parameter that the centralized authority can optimize on. The introduced centralized approach is implemented and solved by a centralized authority, which in a realistic implementation could be the base station or a software defined controller. The corresponding problem is formulated as follows.

$$\max_{\boldsymbol{p},\boldsymbol{x},\boldsymbol{\lambda}} \sum_{k\in K}\left(u_k^O(\boldsymbol{a}_k) + u_k^N(\boldsymbol{a}_k, \boldsymbol{a}_{-k}) - \lambda_kp_k\right)$$
$$\text{s.t. } p_k \in [0, p_k^{max}], \quad x_k \in [0,1] \quad \forall k \in K$$

Let

$$\mathcal{S}(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\lambda}) := \sum_{k\in K}\left(A\log_2(\sigma^2 + h_kp_k(1-x_k))\right) + C\log_2(\sigma^2 + \sum_{k\in K}p_kh_kx_k) - \sum_{k\in K}\lambda_kp_k.$$

Following simple mathematical manipulations, the above optimization problem can be rewritten as below.

$$\textbf{(P1):}\quad \max_{\boldsymbol{p},\boldsymbol{x},\boldsymbol{\lambda}} \mathcal{S}(\boldsymbol{p},\boldsymbol{x},\boldsymbol{\lambda})$$
$$\text{s.t. } p_k \in [0, p_k^{max}],\quad x_k \in [0,1]\ \forall k \in K.$$

Theorem 4: Let $(\boldsymbol{p}^*, \boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ be the maximum of Problem **P1**, and set

$$z_k^* = 1 + \frac{C\sigma^2 - A\sigma^2 - A \cdot \frac{|K|C\sigma^2 - |K|A\sigma^2 + C\sum_{k\in K} h_k p_k^{\max}}{|K|A+C}}{C h_k p_k^{\max}}.$$

Then $p_k^* = p_k^{max}, \lambda_k^* = 0$ and

$$x_k^* = \begin{cases} 0, & \text{if } z_k^* \le 0 \\ z_k^*, & \text{if } z_k^* \in (0,1) \\ 1, & \text{if } z_k^* \ge 1 \end{cases}$$

Proof: Let $\boldsymbol{p}^{\max} := (p_1^{\max}, \ldots, p_{|K|}^{\max})$. Notice that $\mathcal{S}(\boldsymbol{p},\boldsymbol{x},\cdot)$ is decreasing with respect to λ_k; hence we can consider that $\lambda_k^* = 0$, for all $k \in K$. Now notice that $\mathcal{S}(\cdot,\boldsymbol{x},,\boldsymbol{0})$ is increasing with respect to p_k; hence we can consider that $p_k^* = p_k^{\max}$, for all $k \in K$. In other words, the maximum of Problem **P1** is the maximum of the function

$$\mathcal{S} := \mathcal{S}(\boldsymbol{p}^{\max}, \cdot, \boldsymbol{0}).$$

We first compute the critical points of $\mathcal{S}$. We have

$$\frac{\partial \mathcal{S}}{\partial x_k} = \frac{-A h_k p_k^{\max}}{\sigma^2 + h_k p_k^{\max}(1-x_k)} + \frac{C h_k p_k^{\max}}{\sigma^2 + \sum_{k\in K} h_k p_k^{\max} x_k}.$$

Hence $\frac{\partial \mathcal{S}}{\partial x_k} = 0$, for $k = 1, \ldots, |K|$, if and only if

$$C\sigma^2 + C h_k p_k^{\max}(1-x_k) - A\sigma^2 - A\sum_{k\in K} h_k p_k^{\max} x_k = 0. \tag{16}$$

for all $k = 1, \ldots, |K|$. Notice that Eq. 16 is equivalent to

$$x_k = 1 + \frac{C\sigma^2 - A\sigma^2 - A\sum_{k\in K} h_k p_k^{\max} x_k}{C h_k p_k^{\max}}, \tag{17}$$

for all $k = 1, \ldots, |K|$. If we add the $|K|$ equations in Eq. 16, we have

$$\sum_{k\in K} h_k p_k^{\max} x_k = \frac{|K|C\sigma^2 - |K|A\sigma^2 + C\sum_{k\in K} h_k p_k^{\max}}{|K|A + C} \tag{18}$$

and therefore Eq. 17 yields that the critical points of $\mathcal{S}$ are:

$$z_k^* = 1 + \frac{C\sigma^2 - A\sigma^2 - A \cdot \frac{|K|C\sigma^2 - |K|A\sigma^2 + C\sum_{k\in K} h_k p_k^{\max}}{|K|A+C}}{C h_k p_k^{\max}}, \tag{19}$$

for $k \in K$. Now notice that, for each $k \in K$, we have

$$\frac{\partial^2 \mathcal{S}}{\partial x_k^2} = \frac{-A(h_k p_k^{\max})^2}{(\sigma^2 + h_k p_k^{\max}(1-x_k))^2} + \frac{-C(h_k p_k^{\max})^2}{(\sigma^2 + \sum_{k\in K} h_k p_k^{\max} x_k)^2}.$$

Hence $\frac{\partial^2 \mathcal{S}}{\partial x_k^2} < 0$, or, in other words, $\mathcal{S}$ is a coordinate-wise concave function. Hence $x_k^* = z_k^*$ if $z_k^* \in (0,1)$, $x_k^* = 1$, if $z_k^* \ge 1$, and $x_k^* = 0$, if $z_k^* \le 0$. ■

It should be mentioned that the centralized power allocation outcome can be used for benchmarking purposes as well. In particular, in section VII-B, we compare the centralized power allocation outcome to the corresponding ones achieved by the distributed resource allocation under the normal form games. Furthermore, we examine whether setting $\lambda_k = 0,\ \forall k \in K$, as the social welfare optimal solution does, is beneficial for the system and the users. Finally, it is noted that the centralized approach assumes that a centralized authority knows a priori all the user specific information, and that the latter will accept and follow the strategies that they will be assigned to them.

VI. Dual Access Technology Option as a Satisfaction Form Game

A novel approach that can be employed in order to achieve the objective of satisfying the users' minimum QoS requirements involves games in satisfaction form. As mentioned earlier in the paper, a game in satisfaction form is formulated as $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$, where $K = \{1, \ldots, |K|\}$ represents the set of users and A_k is the set $[0, p_k^{\max}] \times [0,1]$, whose elements are denoted $\boldsymbol{a}_k = (p_k, x_k)$. The difference between the G and $\hat{G}$ is that the third component of $\hat{G}$ is not given by the user's utility function, but is given by the function f_k that maps vectors $\boldsymbol{a}_{-k} \in A_{-k}$ to a *set* of vectors $\boldsymbol{a}_k \in A_k$. Specifically, if we denote the minimum QoS requirement of user k as t_k, then f_k is given by

$$f_k(\boldsymbol{a}_{-k}) = \{\boldsymbol{a}_{-k} \in A_k : u_k(\boldsymbol{a}_k, \boldsymbol{a}_{-k}) \ge t_k\}.$$

The function $u_k(\boldsymbol{a}_k, \boldsymbol{a}_{-k})$ represents player k's payoff (i.e., utility function) that is given by

$$u_k(p_k, x_k; \boldsymbol{p}_{-k}, \boldsymbol{x}_{-k}) = u_k^O(p_k, x_k) + u_k^N(\boldsymbol{p}, \boldsymbol{x}) \tag{20}$$

Note, that the only difference between Eq. 20 and Eq. 5 is that the cost function is omitted here.

The rationale of this type of modeling in the case of satisfaction form games, is that the users intrinsically behave in a more social manner by aiming to fulfill their minimum QoS prerequisites, rather than maximizing their utility. Thus, the system has less incentives to penalize them with a usage-based pricing scheme, as in Eq. 5. Furthermore, by adopting the games in satisfaction form, we implicitly assume that the key objective of a user is to reach a particular threshold in its utility function. Therefore, each user has inelastic QoS prerequisites (i.e., Shannon capacity) that even if one explicitly penalized their transmissions, they would be unaffected by it, in case they were still satisfied. This discussion connotes the fact that in the respective optimization problem of a single user, the utility function should be a part of the constraints rather than the optimization objective. Nonetheless, as the definition of ESE points suggests, when players are to choose between some strategies that satisfy them, they will choose the one with the lowest cost. Aligned with the analysis of the previous section, we consider cost functions that are increasing with respect to the transmission powers. Since the user utilizes the cost function only to compare strategies, we can use as cost function the transmission powers itself without loss of generality.

Formulating the problem with a satisfaction form game, we aim at establishing the existence of at least one ESE described in section II-B, along with showing that the system will eventually converge to such an equilibrium.

A. Efficient SE and Best Response Dynamics

In this section, we provide sufficient and necessary conditions that guarantee the existence of an ESE in game $\hat{G}$ and, provided that such an ESE exists, we additionally show that the BRD converge to such an equilibrium. Finally, we employ optimization techniques in order to find the possible BRs of the users, something that lays the foundation for the design of an efficient distributed algorithm. For that purpose, we should first redesign the BR function of each user according to its motives, taking into account the objective of QoS satisfaction. In particular, since players demand to meet their minimum QoS requirements with the lowest possible cost (transmission power) as dictated by the definition of ESE, then given $\boldsymbol{a}_{-k}$, we have that the BR of player k in the satisfaction form game is defined as $\mathcal{BR}_k(\boldsymbol{a}_{-k}) = \{\boldsymbol{a}_k = (p_k, x_k) \in A_k : \boldsymbol{a}_k = \arg\min_{\boldsymbol{a}_k \in f_k(\boldsymbol{a}_{-k})} c_k(p_k)\}$.

Assumption 5: Each user $k \in K$ has a non-empty BR set when the rest of the users choose a BR strategy. That is:

- $\exists a^*_{|K|} \in A_{|K|} : a^*_{|K|} \in \mathcal{BR}_{|K|}(\cdot)$.
- $\exists a^*_{|K|-1} \in A_{|K|-1} : a^*_{|K|-1} \in \mathcal{BR}_{|K|-1}(a^*_{|K|})$.
- $\ldots$
- $\exists a^*_1 \in A_1 : a^*_1 \in \mathcal{BR}_1(\boldsymbol{a}^*)$.

Observe that a vector $\boldsymbol{a}^* = (\boldsymbol{a}^*_1, \ldots, \boldsymbol{a}^*_{|K|})$, given by Assumption 5, is an ESE of the game $\hat{G}$. Using a similar argument as the one in Proposition 1, we can show that the BRD of the game $\hat{G}$ converges to an ESE $\boldsymbol{a}^*$. Later on, we prove that the assumption above is a sufficient and necessary condition for the existence of an ESE in the game $\hat{G}$.

The corresponding optimization problem that arises when each user attempts to find its BR is formulated as follows.

$$\textbf{(P2):} \quad \min_{(p_k, x_k)} \; p_k$$
$$\text{s.t. } u_k(p_k, x_k; \boldsymbol{p}_{-k}, \boldsymbol{x}_{-k}) \geq t_k,$$
$$p_k \in [0, p_k^{\max}] \;\text{ and }\; x_k \in [0, 1].$$

We employ the Karush–Kuhn–Tucker (KKT) conditions in order to determine potential local optima of Problem **P2**. Before proceeding with the details of obtaining the solution, we first provide some remarks about local optima of Problem **P2** as well as the ESEs of $\hat{G}$.

Lemma 6: Suppose that (p_k^*, x_k^*) is a local minimum of Problem **P2**. Then it holds true that $u_k(p_k^*, x_k^*) = t_k$. Furthermore, if (p_k^*, x_1) and (p_k^*, x_2) are both local minima of Problem **P2** then $x_1 = x_2$.

Proof: To prove the first statement, suppose that $u_k(p_k^*, x_k^*) > t_k$ holds true. Then the continuity of u_k, and the fact that $u_k(\cdot, x_k^*)$ is increasing, imply that there exists $\varepsilon > 0$ such that $u_k(p_k^* - \varepsilon, x_k^*) \geq t_k$, contrariwise to the optimality of (p_k^*, x_k^*). To prove the second statement, suppose that (p_k^*, x_1) and (p_k^*, x_2) are local optima of Problem **P2** and assume, without loss of generality, that $x_1 < x_2$. Notice that the function $u_k(p_k^*, \cdot)$ is strictly concave, which in turn implies that for all $x \in (x_1, x_2)$, we have $u_k(p_k^*, x) > t_k$. Let $x_0 \in [x_1, x_2]$ be such that $u_k(p_k^*, x_0) \geq u_k(p_k^*, x)$, for all $x \in (x_1, x_2)$ and let $\varepsilon > 0$ be such that $\varepsilon < u_k(p_k^*, x_0) - t_k$. The intermediate value theorem implies that there exists $x \in (x_1, x_2)$, such that $u_k(p_k^*, x) = t_k + \varepsilon$. Then, since the function $u_k(\cdot, x)$ is increasing, continuous and satisfies $u_k(0, x) = 0$, it follows that there exists $\delta > 0$ such that $u_k(p_k^* - \delta, x) = t_k$, contrariwise to the optimality of (p_k^*, x_1). ■

Lemma 6 implies that for each user $k \in K$ and each $\boldsymbol{a}_{-k} \in A_{-k}$, the set $\mathcal{BR}_k(\boldsymbol{a}_{-k})$ will either be empty or will consist of just one element, namely, $(p_k^*, x_k^*) \in A_k$. In particular, this means that the optimum choice of transmission power corresponds to a unique optimum choice of x_k, and that for every other choice of x_k the utility decreases. That said, whereas we modeled each user's BR to depend on its power consumption and be indifferent of its choice of x_k, it turns out that there is only one x_k^* that corresponds to the optimal solution of k. Moreover, the following proposition proves the uniqueness of the ESE point in $\hat{G}$.

Proposition 7: Assumption 5 $\iff$ There exists an ESE in game $\hat{G}$.

Proof: ($\Rightarrow$) As is already mentioned, the strategy profile $\boldsymbol{a}^*$ that is derived from Assumption 5 is an ESE for $\hat{G}$ and is unique based on Lemma 6.

($\Leftarrow$) Let a random ESE of $\hat{G}$, $\boldsymbol{a}'$. Then we have that $a'_{|K|} \in \mathcal{BR}_{|K|}(\cdot)$ and based of Lemma 6 we have that $a'_{|K|} = a^*_{|K|}$. For user $|K| - 1$ we have that $a'_{|K|-1} \in \mathcal{BR}_{|K|-1}(\boldsymbol{a}'_{|K|}) = \mathcal{BR}_{|K|-1}(\boldsymbol{a}^*_{|K|})$ and from the assumption 5 that $a^*_{|K|-1} \in \mathcal{BR}_{|K|-1}(\boldsymbol{a}^*_{|K|})$. So again, based on Lemma 6, we have that $a'_{|K|-1} = a^*_{|K|-1}$. With the same fashion, we can prove that $a'_k = a^*_k, \forall k \in K$ and thus $\boldsymbol{a}' = \boldsymbol{a}^*$. ■

Corollary 8: When assumption 5 holds true there exists a unique ESE $\boldsymbol{a}^*$ in $\hat{G}$. When assumption 5 does not hold true then there does not exists any ESE in the game $\hat{G}$.

The corollary above states that the Assumption 5 is necessary and sufficient condition for an ESE to exist. In the following analysis, we determine possible optima of Problem **P2** using the KKT conditions (see [28, Chapter 2]). In order to derive the KKT conditions, we first consider the Lagrangian function of Problem **P2**:

$$\begin{aligned}\mathcal{L}(p_k, x_k; \bar{\mu}) = p_k &+ \mu_1 \cdot (-u_k + t_k) \\ &+ \mu_2 \cdot (-p_k) + \mu_3 \cdot (p_k - p_k^{\max}) \\ &+ \mu_4 \cdot (-x_k) + \mu_5 \cdot (x_k - 1). \qquad (21)\end{aligned}$$

The KKT theorem (see [28, Theorem 2.1]) states that if (p_k^*, x_k^*) is a local minimum of Problem **P2** then, there exists a non-zero vector of Lagrange multipliers, which satisfies the following conditions.

Condition 9 (KKT conditions): If (p_k^*, x_k^*) is a local minimum of Problem **P2** then there exists a vector $(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5) \neq \mathbf{0}$ such that:

1) $-\mu_1 \cdot \frac{\partial u_k(p_k^*, x_k^*)}{\partial x_k} - \mu_4 + \mu_5 = 0$
2) $1 - \mu_1 \cdot \frac{\partial u_k(p_k^*, x_k^*)}{\partial p_k} - \mu_2 + \mu_3 = 0$
3) $\mu_1 \cdot (-u_k(p_k^*, x_k^*) + t_k) = 0$
4) $\mu_2 \cdot (-p_k^*) = 0$
5) $\mu_3 \cdot (p_k^* - p_k^{\max}) = 0$
6) $\mu_4 \cdot (-x_k^*) = 0$
7) $\mu_5 \cdot (x_k^* - 1) = 0$
8) $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \geq 0$
9) $p_k^* \in [0, p_k^{\max}]$
10) $x_k^* \in [0, 1]$.

The parameters $\mu_1, \ldots, \mu_5$ denote the *Lagrange multipliers*. Using the KKT Conditions 9, we obtain the following.

Theorem 10: Suppose that (p_k^*, x_k^*) is a local minimum of Problem **P2**. Then (p_k^*, x_k^*) is one of the following points:

1) $(\frac{\sigma^2(e^{t_k/A}-1)}{h_k}, 0)$
2) $(\frac{(\sigma^2+J_k)(e^{t_k/C}-1)}{h_k}, 1)$
3) $(p_k^{\max}, x_*)$, where $x_* = \frac{C\sigma^2+Ch_k p_k^{\max}-A(\sigma^2+J_k)}{(A+C)h_k p_k^{\max}}$.
4) (p_*, x_*), where

$$p_* = (A+C)\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C h_k^{A+C}}\right)^{\frac{1}{A+C}} - \frac{2\sigma^2+J_k}{h_k}$$

and

$$x_* = \frac{C\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}} - (\sigma^2+J_k)}{(A+C)\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}} - (2\sigma^2+J_k)}$$

In the following, we first discuss some interesting observations regarding the Lagrange multipliers. Assume that (p_k^*, x_k^*) is a local minimum of Problem **P2**. Lemma 6 implies that $u_k(p_k^*, x_k^*) = t_k$ must hold true, and hence Condition 9.3 implies that $\mu_1 > 0$. We now claim that we can assume that the Lagrange multipliers satisfy $\mu_2 = \mu_3 = \mu_4 = \mu_5 = 0$. Indeed, since $t_k > 0$, we have $p_k^* > 0$ and thus Condition 9.4 implies that $\mu_2 = 0$. Condition 9.5 implies that if $\mu_3 > 0$ then $p_k^* = p_k^{\max}$, which in turn implies that x_k^* must satisfy $u_k(p_k^{\max}, x_k^*) = t_k$. Let x_m be the smallest $x \in [0,1]$ such that $u_k(p_k^{\max}, x_m) = t_k$ and let x_M be the largest $x \in [0,1]$ such that $u_k(p_k^{\max}, x_m) = t_k$. From Lemma 6 it follows that $x_m = x_M$ and therefore, since $u_k(p_k^{\max}, \cdot)$ is concave, we have that x_k^* must satisfy $\frac{\partial u_k(p_k^{\max}, x_k^*)}{\partial x_k} = 0$ or, equivalently, based on Eq. 7, that $x_k^* = \frac{C\sigma^2+Ch_k p_k^{\max}-A(\sigma^2+J_k)}{(A+C)h_k p_k^{\max}}$.

Condition 9.6 implies that if $\mu_4 > 0$ then $x_k^* = 0$ and therefore, since u_k is increasing with respect to p_k, for fixed x_k, it follows that $p_k^* = \frac{\sigma^2(e^{t_k/A}-1)}{h_k}$, provided $\frac{\sigma^2(e^{t_k/A}-1)}{h_k} \in [0, p_k^{\max}]$. For the same reason, Condition 9.7 implies that if $\mu_4 > 0$ then $x_k^* = 1$ and hence $p_k^* = \frac{(\sigma^2+J_k)(e^{t_k/C}-1)}{h_k}$, provided $\frac{(\sigma^2+J_k)(e^{t_k/C}-1)}{h_k} \in [0, p_k^{\max}]$. Summarising the discussion, we conclude to the following.

Corollary 11: Let (p_k^*, x_k^*) be a local minimum of Problem **P2**. Then the following hold true:

1) If $x_k^* = 0$ then $p_k^* = \frac{\sigma^2(e^{t_k/A}-1)}{h_k}$.
2) If $x_k^* = 1$ then $p_k^* = \frac{(\sigma^2+J_k)(e^{t_k/C}-1)}{h_k}$.
3) If $p_k^* = p_k^{\max}$ then $x_k^* = \frac{C\sigma^2+Ch_k p_k^{\max}-A(\sigma^2+J_k)}{(A+C)h_k p_k^{\max}}$.

We may therefore assume that $\mu_2 = \mu_3 = \mu_4 = \mu_5 = 0$ and we are looking for a triplet $(p_k^*, x_k^*, \mu_1) \in (0, p_k^{\max}) \times (0,1) \times (0,\infty)$ that satisfies Condition 9.1–3. This is the content of the following result.

Proposition 12: There is a unique triplet (p_k^*, x_k^*, μ_1), that satisfies Condition 9, 1–3, and the coordinates are given by

$$x_k^* = \frac{C\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}} - (\sigma^2+J_k)}{(A+C)\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}} - (2\sigma^2+J_k)},$$

$$p_k^* = \frac{A+C}{h_k}\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}} - \frac{2\sigma^2+J_k}{h_k},$$

and

$$\mu_1 = \frac{1}{h_k}\left(\frac{e^{t_k}\sigma^{2A}(\sigma^2+J_k)^C}{A^A C^C}\right)^{\frac{1}{A+C}}.$$

The proof of Proposition 12 is deferred to the Appendix B. Bearing in mind the aforementioned analysis, we proceed with completing the proof of Theorem 10.

Proof of Theorem 10: Let (p_k^*, x_k^*) be a local minimum of Problem **P2**. Then Corollary 11 and Proposition 12 imply that (p_k^*, x_k^*) is either one of the first three points in Theorem 10 or, provided its coordinates belong to the set $(0, p_k^{\max}) \times (0,1)$, the fourth point in Theorem 10. ■

In the following analysis we provide sufficient conditions that ensure that the BR does not lie at the boundaries. We clarify here that such conditions are not necessary for the convergence of the BRD (and subsequently for the convergence of the algorithm proposed in the following section) to the ESE. Notice that in principle, the fourth point in Theorem 10 may not belong to the set $(0, p_k^{\max}) \times (0,1)$. Indeed, the following hypothesis provides sufficient conditions that guarantee that it does belong to this set.

Assumption 13: Let $S := \left(\frac{A^A C^C}{\sigma^{2A}(\sigma^2+J_k)^C}\right)^{\frac{1}{A+C}}$. A threshold, t_k, in Problem **P2** will be referred to as *special*, if it satisfies the following inequalities:

$$t_k > (C+A)\cdot\log_2\left(S\cdot\frac{2\sigma^2+J_k+2^{-1/4}h_k^{1/2}(\sigma^2+J_k)^{1/2}}{C+A}\right) \tag{22}$$

and

$$t_k < (C+A)\cdot\log_2\left(S\cdot\frac{2\sigma^2+J_k+h_k p_k^{\max}}{C+A}\right). \tag{23}$$

Furthermore, t_k is assumed to satisfy

$$t_k \geq (C+A)\cdot\log_2\left(S\cdot\frac{2\sigma^2+J_k}{A}\right). \tag{24}$$

In particular, notice that in the assumptions stemming from Eq. 22, Eq. 23 and Eq. 24, we implicitly consider that

$$2^{-1/4}(\sigma^2+J_k)^{1/2} < h_k^{1/2}p_k^{\max}$$

and

$$\frac{2\sigma^2+J_k}{A} < \frac{2\sigma^2+J_k+h_i p_k^{\max}}{C+A}$$

Lemma 14: Suppose that t_k is a special threshold, as defined in Assumption 13. Then the triplet (p_k^*, x_k^*, μ_1), given by Proposition 12, additionally satisfies $(p_k^*, x_k^*, \mu_1) \in (0, p_k^{\max}) \times (0,1) \times (0,\infty)$.

Proof: Now notice that Assumption 13 implies that $p_k^* \in (0, p_k^{\max})$. Moreover, it follows from Eq. 24 that $x_k^* > 0$ as well as $x_k^* < 1$. Hence, provided Assumption 13 holds true, we have $(x_k^*, p_k^*) \in (0,1) \times (0, p_k^{\max})$. ■

Furthermore, we report the fact that Assumption 13 is sufficient for the triplet provided by Proposition 12 to be a local optimum of Problem **P2**. More precisely, we have the following proposition.

Proposition 15: Suppose that Assumption 13 holds true. Then the pair (p_k^*, x_k^*), whose coordinates are defined in Proposition 12, is a local minimum of Problem **P2**.

The proof of Proposition 15 is deferred to the Appendix C.

Algorithm 2 Best Response for Satisfaction (BRS)

$\mu \leftarrow \infty$; // Represents the minimum power
$\mathcal{D} \leftarrow \emptyset$; // Represents the minimizers, given $\boldsymbol{a}_{-k}$. $\mathcal{D}.p$ denotes the power and $\mathcal{D}.x$ the splitting factor
$\boldsymbol{\delta} \leftarrow \emptyset$;
for $\boldsymbol{\delta}$ `from points 5.8.1 to 5.8.4` **do**
  if $\boldsymbol{\delta} \in [0, p_k^{max}] \times [0,1]$ **then**
    if $U_k(\boldsymbol{\delta}, \boldsymbol{a}_{-k}) \geq t_k$ **then**
      if $\delta.p < \mu$ **then**
        $\mathcal{D} \leftarrow \boldsymbol{\delta}$;
        $\mu \leftarrow \delta.p$;
      end if
    end if
  end if
end for
play $\mathcal{D}$; // Play the Best Response

B. Best Response for Satisfaction (BRS) Algorithm

Following the analysis of the proof of Theorem 10, we provide the *Best Response for Satisfaction* algorithm that should be executed by each user k who knows the strategies of the other users (the knowledge of the strategies of the users with worse channel gains is sufficient). Each user should initialize at first (randomly) its choice while after executing the BRS algorithm it will have chosen its BR. Similar to Section IV-A, if the users are ordered according to their respective distances from the BS, each user should execute BRS only once, and thereby $|K|$ turns of the algorithm are sufficient for the system to converge to the ESE. Similar to the BRM algorithm and the dynamics that it dictates, BRS needs $\mathcal{O}(1)$ time complexity to determine the BR of a user, while the total dynamics would need $\mathcal{O}(|K|)$ in such scenario.

VII. Numerical Results

In this section, we provide a detailed numerical performance evaluation of the proposed power allocation framework through modeling and simulation, illustrating the operation, features and benefits of the proposed modeling and approaches. Specifically, in Section VII-A, we initially establish the path - through the use of different pricing scenarios - so that centralized authority could manipulate the NE point at which the users tend to converge, in order to make the overall power allocation more fair and efficient. Subsequently, in Section VII-B the NEs resulting from the distributed power allocation (i.e., games in normal form) under the employment of different pricing scenarios, are compared against the corresponding centralized solution (of Section V). Moreover, in section VII-C, we examine the benefits of the framework of satisfaction form games when used as a mechanism to formulate and solve the power allocation problem under consideration. In particular, we compare the outcome of the NE based solution, with the corresponding ESE points resulting from the game in satisfaction form for $three$ different indicative scenarios of users' QoS prerequisites. In our study, for demonstration purposes, we consider $|K| = 25$ users in the system distributed in an equal step distance from the base station, ranging from $40m$ to $1Km$. The bandwidth of each

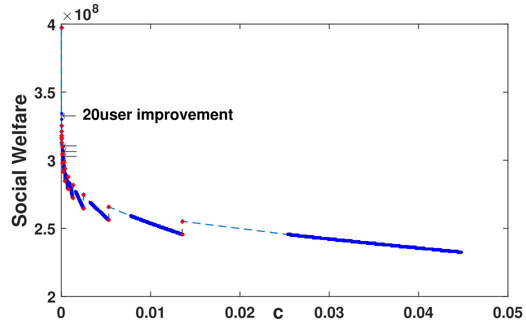


Fig. 1. Social Welfare as a function of the pricing factor c.

OFDMA channel is $A = 180kHz$ and we consider 25 orthogonal such channels totaling a bandwidth of approximately 5Hz, while the total shared bandwidth among all the users in the NOMA band is $C = 10MHz$. The users' maximum transmission power is assumed to be $p_k^{max} = 2W, \forall k \in K$.

A. Pricing Considerations and Nash Equilibrium Points

The concept of resource pricing has been used in the literature in order to enable the distributed resource management approaches to conclude to more efficient outcomes from a social welfare point of view. In our case, the usage-based pricing scheme $c_k(p_k) = \lambda_k p_k$, adopted in Eq. 5, depends on the user's personal transmission power p_k. In principle, the role of the pricing parameter λ_k of each user k is twofold. First, it can be used to express the personalized dissatisfaction of the user regarding the consumption of its own personal resource, i.e. the mobile device's available battery, thus, motivating the user to sparingly use its limited energy resources. Second, it acts as a control parameter by the centralized authority to drive the overall system in a more desirable operation state. Our evaluation, focuses on the latter, and thus in the following, we particularly examine the role of pricing, under the perspective that λ_k is determined by the network provider (i.e., centralized authority).

For simplicity in the presentation and subsequent discussion, we set $\lambda_k = c\alpha_k$, where c is a constant pricing factor that is imposed to every user, while α_k is a personalized pricing factor dependent on the user k. It should be reminded here that, as shown in section V, through the centralized approach the optimal solution of the power allocation problem from the perspective of the social welfare, occurs when $\lambda_k = 0$, for all $k \in K$. In this section, we experiment with the constant pricing factor c in order, on the one hand to gain some insight about the impact of the socially optimal zero-cost pricing on various performance criteria such as fairness, etc., while on the other hand to propose the most appropriate value for c (in the following we refer to as c^{best}), that drives the system to operate in a desirable state, as detailed below.

Fig. 1 presents the resulting social welfare (summation of the corresponding user utilities) for the solution of the distributed resource allocation as a function of the pricing factor c. This is obtained by calculating the corresponding social welfare achieved when the users converge to the respective NE, while considering each time an arbitrary fixed value for the parameter c. We also note that, though the procedure described here is fundamentally different from the solution

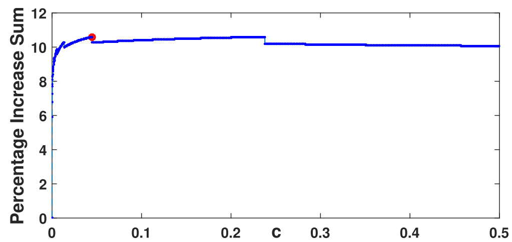


Fig. 2. Percentage increase summation of the user utilities wrt the social welfare optimal solution ($c = 0$), vs. the pricing factor c.

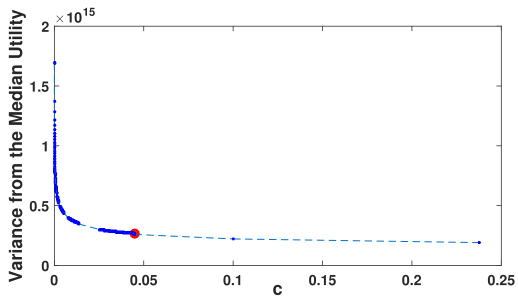
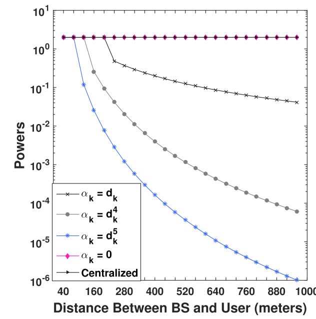


Fig. 3. Variance of utility from the median with respect to c.

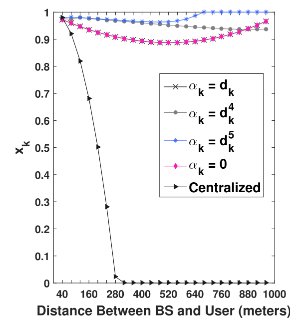
of the centralized resource allocation problem in Section V, the maximum social welfare, is also achieved for zero cost, i.e., $c = 0$, thus, $\lambda_k = 0, \forall k \in K$. This can be also proven formally, if one solves the corresponding maximization problem over $\boldsymbol{\lambda}$ of the summation of the utility functions given the NE, which depends on the $\boldsymbol{\lambda}$ itself. In other words the NE with the maximum social welfare corresponds to $c = 0$. However, for increasing c we note in Fig. 1 that multiple local extrema (red points) may occur. An additional interesting observation is that there are some choices of the cost parameter c (black arrows), where for instance 20 out of 25 users result in greater utilities than in the corresponding solution under the objective of social welfare optimality (i.e. $c = 0$).

The latter observation motivates the need for investigating more in depth the impact and benefit of different pricing factors on various user and network performance metrics of high interest and importance, other than solely the social welfare. To better quantify this, let u_k^0 be the utility of the kth user under the social welfare optimal solution (i.e. $c = 0$) and u_k^c be the utility that the NE allocates to user k when the fixed cost factor c is used. Then, for each c we examine the quantity $\sum_{k\in K}(u_k^c - u_k^0)/u_k^0$ which is the summation of the percentage increases of the utilities of the users. Fig. 2, shows that this metric increases as we increase c until a point (red circle) - reflecting c^{best} - where, after this, the metric is strictly lower. At this point the aforementioned quantity reaches the value of 10.60 corresponding on average to a 42.4% increase on each user utility. Similar observations and conclusions can be drawn from Fig. 3 where we notice that increasing c, reduces the variance, $\sum_{k\in K}(u_k^c - median)^2/|K|$, from the median utility at a specific c, while the value c^{best} is very close to the "knee" of the graph after which the decrease on the variance begins to saturate.

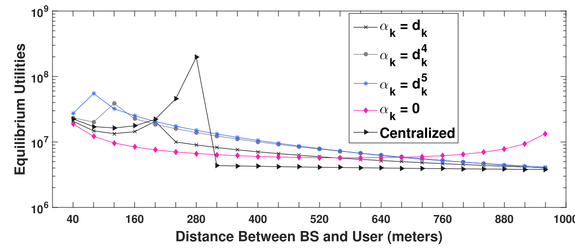
With reference to the adopted pricing parameter $\lambda_k = c\alpha_k$, apart from the fixed factor c discussed above, it is also noted that the pricing factor α_k - being in nature different for each user k - can be further exploited, and in particular in the



(a) Power Allocation



(b) Splitting Factors (x_k)



(c) Utilities at equilibrium

Fig. 4. Power allocation, splitting factors and utilities at equilibrium for each user (i.e. function of user distance from the BS), for various pricing scenarios ($\alpha_k = d_k^j$).

case of adopting NOMA. For that purpose, in the comparative results presented in the following subsection, we examine different pricing factors α_k that depend on the distance d_k of user k from the BS, in the following form $\alpha_k(d_k) = d_k^j$, where the exponent j is an integer allowing the realization of different pricing scenarios (from less strict to more aggressive ones). For completeness we note, that for the results that have been presented so far, for demonstration only purposes, were obtained using the specific form of $\alpha_k(d_k) = d_k^4$. Taking this into account, it is worth noting that in Fig. 3 as c continues to increase, the variance does not increase whereas some users (those in particular with bad channel gains) obtain utilities that are virtually zero, owing to the pricing factors $\alpha_k(d_k) = d_k^4$ adopted. This is due to the fact that, as it will be also clarified and demonstrated in the next subsection, the users with bad channel gains, obtain large utilities by worsening the NOMA channel conditions in the lower values of λ_k.

B. Comparison Between the NEs and the Centralized Solution

For the remaining of this section we assume the use of $c = c^{best}$ for any given α_k. In particular, Fig. 4 studies the power allocation (Fig. 4a), the corresponding splitting factors x_k (Fig. 4b) and the resulting utilities (Fig. 4c) at the NE for four different pricing factors α_k, as well as for the optimal centralized solution (i.e., Section V). In Fig. 4a- 4c, the purple curves represent the NE values for the scenario of zero cost, i.e., $\alpha_k = 0$, which as demonstrated in the previous subsection concludes to the optimal social welfare as well. On the other hand the remaining three lines correspond to more strict pricing for the users with bad channel gains, and the last curve represents the centralized solution.

By comparing the centralized solution with the other four curves we notice that the central authority forces several users (18 users in our scenario), especially the ones with bad channel gains, to transmit only in the OFDMA channel in order to reduce the interference in the NOMA transmission band (Fig. 4b), thus introducing unfairness for some users in accessing the NOMA available bandwidth. We also observe from Fig. 4c that the utility obtained by those users is very low, as opposed to the users with the lowest distance from the BS. Note, that the users that treat the NOMA channel with lower interference although having low splitting factors, they enjoy higher utilities. Furthermore, comparing the four curves that correspond to the decentralized solutions, we notice that in Fig. 4c when there is no pricing (purple curve), the utilities are significantly higher for the users with very bad channel gains. In the case of zero pricing, we notice that the users with either very good or very bad channel gains have significantly higher utilities than the users with medium channel gains.

Nonetheless, with the use of appropriate pricing schemes, we can spread the benefits stemming from the application of the SIC technology in a more fair manner among the users, instead of solely allowing the users with worse channel gains to inflate the system. Indeed, when the exponent j in distance based pricing factor $\alpha_k = d^j$ is increasing, we can see that we obtain a more desirable curve, that corresponds to an increase in the utility of those users with better channel gains, who do not however increase notably the interference of the system. Nevertheless, for exponents with high values (e.g. greater than 5), the users with high distance from the BS get very low utilities. That said, the network provider, could appropriately select the values of pricing factors α_k towards trading interference, and thus network capacity, and fairness.

From the results in Fig. 4b we observe that, when there is no pricing (purple curve) or when pricing is less aggressive, that is $\alpha_k = d_k$, the splitting factors follow a convex function as users with high and low channel gains invest more in the NOMA channel. On the other hand, when we increase the exponent on the distance in the pricing factors, we see that we motivate users with higher channel gains to invest more in the NOMA channel as users with low channel gains do not increase dramatically the introduced interference in this band. In Fig. $4c$ we notice that both in the decentralized social welfare optimal NE (purple curve) and in the centralized solution (black curve), all users transmit with their highest possible power. Moreover, when the exponent in the pricing factor $\alpha_k(d_k)$ increases, under the operation of the decentralized approach the willingness of the users with lower channel gains to transmit with higher powers decreases as well.

One interesting observation is the occurrence of the spikes in the corresponding utility curves in Fig. 4c, under the decentralized approach when pricing is applied. These spikes occur for the user with the lowest channel gain that however transmits with its maximum transmission power. This is due to the fact that the users with lower distance from the BS have true maximum utilities way beyond the frontier of $[0, p^{max}], [0, 1]$, and they are forced to transmit with p^{max} only because it is their boundary. On the other hand the user with the spiked utility has true maximum that is near its p^{max} (but greater than it) on the p-axis.

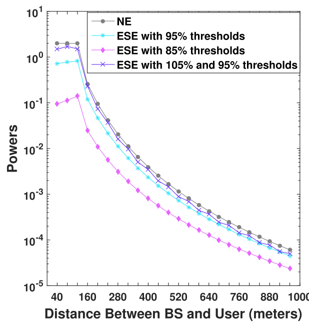
C. Satisfaction Form Game & ESE

In this section we provide a comparative analysis and evaluation of the results obtained via the NE bases solution, for the case where $\alpha_k = d_k^4$ and $c = c^{best}$, against the corresponding outcomes resulting from the framework of games in satisfaction form, as it was analyzed in section VI. For the purposes of the evaluation, the thresholds required for the operation of the framework of the games in satisfaction form were based on the values of the utilities of the specific NE point mentioned above. In particular, three different such scenarios and operation points were studied and compared against the aforementioned NE point. In a nutshell, the three alternative scenarios under consideration are as follows:

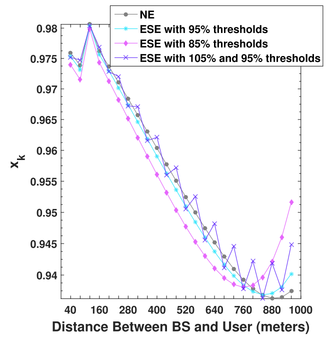
- Each user k has threshold t_k that is 95% of the corresponding utility obtained at the NE.
- Each user k has threshold t_k that is 85% of the corresponding utility obtained at the NE.
- A mixed set of users is utilized, where half of them have thresholds that are 95% of the obtained utilities at the NE, while the remaining present higher thresholds, that are 105% of the obtained utilities at the NE.

Accordingly, Fig. 5, presents the power allocation (Fig. 5a), corresponding splitting factors x_k (Fig. 5b) and the resulting utilities (Fig. 5c) at the NE, and the corresponding ESE points, for the $three$ aforementioned scenarios. In Fig. 5a, we notice that if the users applications required only 85% of the utility gained at the NE (second scenario), the system would be characterized by significantly higher efficiency, as each user would decrease its transmission power. Lower but still notable decrease in transmission powers are also observed even when the users set thresholds at 95% of the corresponding utility of the NE (i.e. first scenario). Furthermore, with respect to the third scenario of mixed set of users, we observe that, even the users that required higher utility thresholds than the corresponding utilities at the NE point, managed to converge to a point that satisfied them. More importantly, the latter happened while all the users in the system managed to decrease their transmission power, no matter if they demanded - and ultimately obtained - either 95% or 105% of the respective utility experienced at the NE. In Fig. 5b we note that all users, located after a specific distance from the BS, increase their transmission power splitting factors in both the ESEs with the lower QoS prerequisites, when compared with the respective splitting factors at the NE point. This is expected and justified, as those users realize that they can satisfy their QoS prerequisites while transmitting with a lower power, only by increasing their investments in the NOMA option.

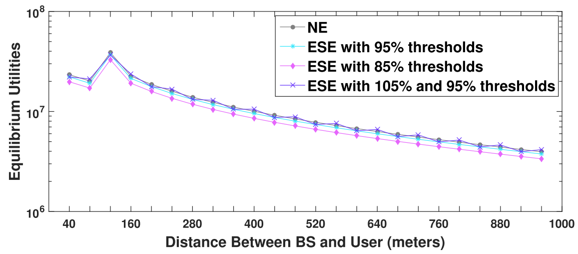
Fig. 6 depicts the interference, as sensed at the receiver, for each user (i.e., as a function of the distance from the BS). As the QoS prerequisites diminish, the interference plummets. Surprisingly, it is noted that when all the users request 85% of the utility earned at the NE (that is a 15% drop in the values of their utilities), the total interference drops by 93.38%, a factor



(a) Power Allocation



(b) Splitting Factors (x_k)



(c) Utilities at equilibrium

Fig. 5. Power allocation, splitting factors and utilities at the ESE for each user (function of user distance from the BS).

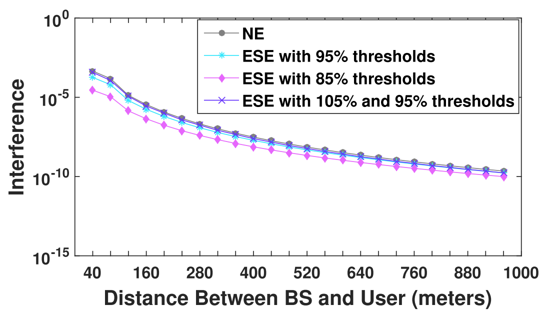


Fig. 6. Interference sensed, for each user, as a function of the user distance from the BS.

that significantly contributes to an increase in the expected capacity of the network.

VIII. Conclusion & Future Work

In this work, we transformed and properly modeled the uplink power allocation problem in order to facilitate the newly emerging 5G dual wireless multiple access paradigm, where both OFDMA-based and NOMA-based transmission options coexist, and users may utilize simultaneously both of them. For that purpose, we initially enriched the mechanism with a Shannon-based utility function towards letting the users choose not only their transmission powers, but also the percentage of their investment in the NOMA-based and/or OFDMA-based transmissions. The resulting multi-variable power allocation problem was treated and solved under three different perspectives, namely: (i) Games in Normal Form and NE, (ii) Optimization techniques targeting system social welfare through a centralized optimal solution, and (iii) Games in Satisfaction Form and ESE.

Based on these approaches, different solutions and stable operation points were obtained, presenting different characteristics and tradeoffs, both from the network provider and the user perspectives. Numerical results achieved via modeling and simulations, allowed the in depth evaluation and comparison of the various obtained outcomes, in terms of the impact and the interplay of SIC specific features, over-exploitation of, and fairness in accessing, the NOMA-based bandwidth, and interference treatment. Finally, in conclusion we confirmed that using the satisfaction form games in order for the users to converge to the ESE, provides a promising and appealing user-centric modeling approach to the power allocation problem, as the system can adapt to the users' application needs, while at the same time eliminates a significant amount of interference.

For setting the foundations of our framework, in this work the basic communications setting that has been considered consists of multiple users and a base station, thus, constructing a single cell networking and communications environment. Our goal is to extend this methodology to more complex networking environments. Such an example would refer to the consideration of a multi-cell environment, where the intracell and intercell interference can be jointly studied in the transmission power allocation problem through the games in satisfaction form. In such a networking environment, problems like users' mobility impact can also be explored. It is interesting to note that, given the operation of NOMA and SIC inherent characteristics, when a user changes its position within the network, thus, its corresponding channel gain conditions change, it will affect the best response strategies of the users with better channel conditions than its own. Capitalizing on that, efficient and effective power allocation strategies within the framework of satisfaction games may be devised.

Finally, as part of our current and future work, within the adopted dual multiple access paradigm, we aim to enrich the proposed holistic power allocation modeling approach, by considering that users in reality present a risk aware behavior. Towards this direction, the OFDMA option may be treated as a safe resource, while the NOMA option can be viewed as a Common Pool Resource, susceptible to failure. To properly model and study the impact of users' and system's behavior on the power allocation problem in this setting, concepts from Prospect Theory and Tragedy of the Commons can be adopted.

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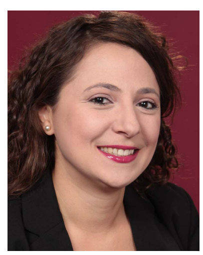

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